Methods for proving identities

General principles

- ✪ Only rewrite each side of the equality – do **NOT** perform equal operations on both sides as if solving an equation
- ✪ Work on the harder side first – typically the side with addition, fractions or more operations
- ✪ Keep in mind what the other side looks like – to determine what terms or factors need to be introduced

Analytic (algebraic) techniques (generally in the following order)
Repeat steps 1, 2, 3 as many times as necessary

0. Use the negative angle identities and cofunction identities

1. Use the Pythagorean identities if there are even powers of trigonometric functions,
especially if both functions from one Pythagorean identity are involved

2. Use the reciprocal and/or quotient identities if there are 3 or more trigonometric functions,
or 2 trigonometric functions which are not related by a Pythagorean identity –
simplify any resulting complex fractions immediately

3. Use general algebraic techniques –
factoring / distributing
performing fraction operations (adding, subtracting, multiplying, dividing, simplifying complex fractions)
adding & subtracting the same quantity
multiplying by a fraction with the same numerator and denominator

Basic identities

Negative angle identities: $\sin(-x) = -\sin x$ $\quad \cos(-x) = \cos x$ $\quad \tan(-x) = -\tan x$

$\csc(-x) = -\csc x$ $\quad \sec(-x) = \sec x$ $\quad \cot(-x) = -\cot x$

Cofunction identities: $\cos x = \sin(\frac{\pi}{2} - x)$ $\quad \csc x = \sec(\frac{\pi}{2} - x)$ $\quad \cot x = \tan(\frac{\pi}{2} - x)$

$\sin x = \cos(\frac{\pi}{2} - x)$ $\quad \sec x = \csc(\frac{\pi}{2} - x)$ $\quad \tan x = \cot(\frac{\pi}{2} - x)$

Pythagorean identities: $\sin^2 x + \cos^2 x = 1$ $\quad \sin^2 x = 1 - \cos^2 x$ $\quad \cos^2 x = 1 - \sin^2 x$

$\tan^2 x + 1 = \sec^2 x$ $\quad \tan^2 x = \sec^2 x - 1$ $\quad \sec^2 x - \tan^2 x = 1$

$\cot^2 x + 1 = \csc^2 x$ $\quad \cot^2 x = \csc^2 x - 1$ $\quad \csc^2 x - \cot^2 x = 1$

Reciprocal identities: $\csc x = \dfrac{1}{\sin x}$ $\quad \sec x = \dfrac{1}{\cos x}$ $\quad \cot x = \dfrac{1}{\tan x}$

$\sin x = \dfrac{1}{\csc x}$ $\quad \cos x = \dfrac{1}{\sec x}$ $\quad \tan x = \dfrac{1}{\cot x}$

Quotient identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\quad \cot x = \dfrac{\cos x}{\sin x}$